Probability

"The chances of occurrence of an event" is called probability"

Subjective probability
⇓
Based on opinions or belief

objective probability
⇓
Based on recorded observations (Data)

Probability of an event is denoted by $P(E)$

$$\boxed{0 \le P(E) \le 1}$$

$$\boxed{P(E) + P(\text{Not } E) = 1}$$

$$\boxed{P(\text{Not } E) = 1 - P(E)}$$

Random experiment : Any experiment which can produce more than one possible outcomes & where the result can not be predicted

eg Tossing a coin

eg Throwing Two dice together

Sample space : set (collection) of all possible outcomes which are associated to a random experiment

eg coin Tossed once
$S = \{H, T\}$

eg Dice is thrown once
$S = \{1, 2, 3, 4, 5, 6\}$

Event : It is a subset of sample space

-> simple event : It contains only one element

-> compound event : It contain more than one element

eg A coin is tossed Twice
$S = \{HH, HT, TH, TT\}$

$E = \{HH\}$
$F = \{HT\}$
simple events

$G = \{HH, HT\}$
$H = \{HH, TT, TH\}$
compound events

→ Impossible event (Empty) : It can never happen

$P(E) = 0$

→ Sure event : It will definitely happen

$P(E) = 1$

eg $S = \{1, 2, 3, 4, 5, 6\}$

E: A number less then 7 $= \{1, 2, 3, 4, 5, 6\}$

Sure event

F: A number more then 7 $= \{\} = \phi$

Impossible event

→ Equally likely events

Two or more events with same probability

eg coin is tossed

$P(H) = \frac{1}{2}$ & $P(T) = \frac{1}{2}$

eg Dice is thrown

$P(1) = P(2) = \cdots P(6) = \frac{1}{6}$

→ complimentary events

Not happening of an event is known as complimentary event

Denoted by E' or $\bar{E}$

$$P(E) + P(\bar{E}) = 1$$

eg A coin tossed Twice

$S = \{HH, HT, TH, TT\}$

E: Two Heads = $\{HH\}$

E' or $\bar{E} = \{HT, TH, TT\}$

↳ This is compliment of E

→ $\boxed{\text{mutually exclusive events}}$

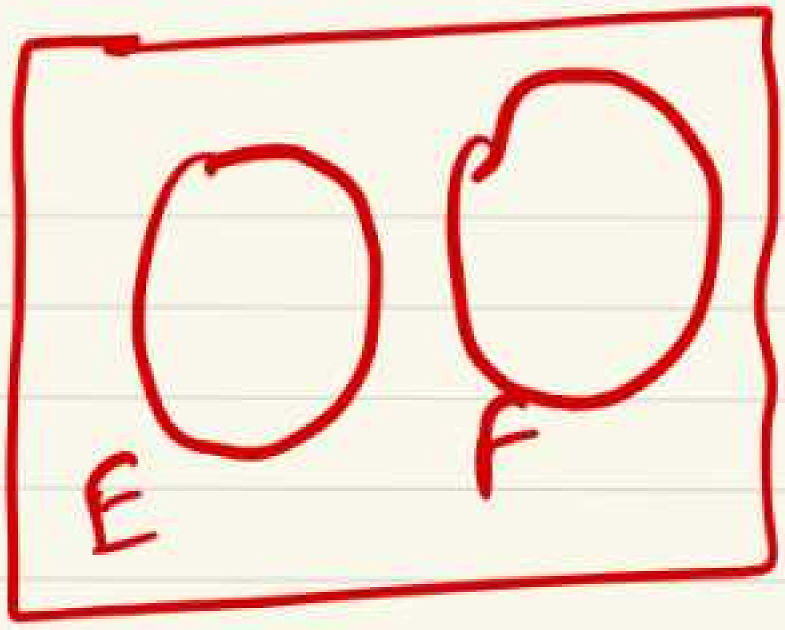
when selection of one event results rejection of the events

or

when two events can not happen simultaeneously

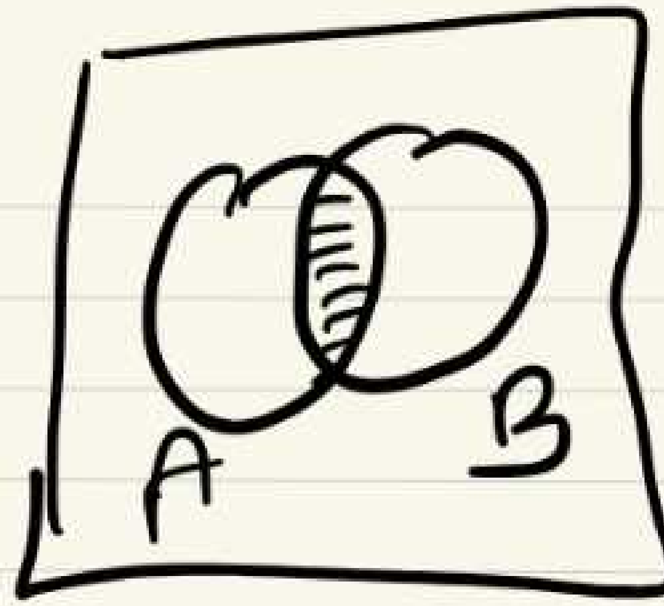
or

when two events have nothing in common



$E \cap F = \phi$

Mutually exclusive



$A \cap B \neq \phi$

Not m. exclusive

eg A: Aman is voter of Delhi
B: Aman is 14 years old

m. exclusive

eg E: Anita is good singer
F: Anita speaks fluent english

Not m. exclusive

If two events are m. exclusive
Then $E \cap F = \phi$

$P(E \cap F) = 0$

→ **Mutually exhaustive events**

Two or more events are mutually exhaustive if their union makes sample space

i.e. $A \cup B = S \Rightarrow \boxed{P(A \cup B) = 1}$

eg $S = \{1, 2, 3, 4, 5, 6\}$

A: Even $= \{2, 4, 6\}$

B: odd $= \{1, 3, 5\}$

$A \cup B = S$ m. exhaustive

E: Prime $= \{2, 3, 5\}$

F: odd $= \{1, 3, 5\}$

$A \cup B = \{1, 2, 3, 5\} \neq S$

Not m. exhaustive

Classical (Priori) Definition of Probability

<u>Assumes</u>

- → All outcomes are known
- → All outcomes are equally likely
- → all outcomes are mutually exclusive

$$P(E) = \frac{\text{Total no of favorable outcomes}}{\text{Total No. of Possible outcomes}}$$

eg $S = \{1, 2, 3, 4, 5, 6\}$ → Dice

$$P(\text{even}) = \frac{3}{6} = \frac{1}{2}$$

$\boxed{\text{Relative frequency Definition of Probability}}$

⇓

Statistical Definition

$n \Rightarrow$ No. of times an experiment is repeated

$n \longrightarrow \infty$ (n is a big number)

$f_A \Rightarrow$ An event 'A' is repeated f_A times

Then Probability of event A

$$P(A) = \lim_{n \to \infty} \frac{f_A}{n}$$

Axiomatic (modern) Definition of Probability

Axiom : Any Rule which don't require proof

under This axiomatic approach

→ P(A) is always non negative

$$P(A) \geq 0$$

→ Prob. of sample space is a sure event

i.e. $P(S) = 1$

→ If A & B are m. exclusive Then $P(A \cup B) = P(A) + P(B)$

Odds in Favour & Against

Odds in favour of $A = A : \bar{A}$

odds against of $A = \bar{A} : A$

$$P(A) = \frac{A}{A+\bar{A}} \quad \& \quad P(\bar{A}) = \frac{\bar{A}}{A+\bar{A}}$$

Q odds in favour of A solving a problem is $3:5$

$$P(A) = \frac{3}{8} \quad \& \quad P(\bar{A}) = \frac{5}{8}$$

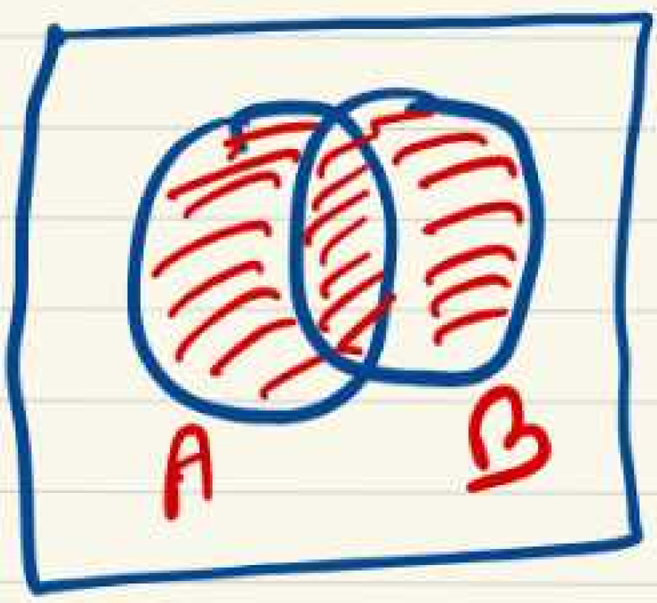
Q odds against A solving a problem is $4:5$

$$P(A) = \frac{5}{9} \quad \& \quad P(\bar{A}) = \frac{4}{9}$$

Set Theory in Probability

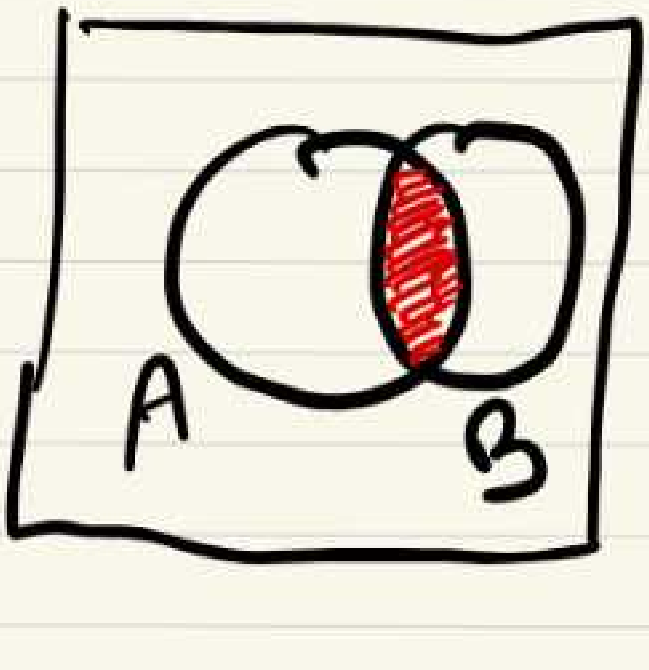
$P(A \text{ or } B) = P(A \cup B) = P(\text{Atleast one set})$

$$= P(A) + P(B) - P(A \cap B)$$



$P(A \,\&\, B) = P(A \cap B)$

$$= P(A) + P(B) - P(A \cup B)$$



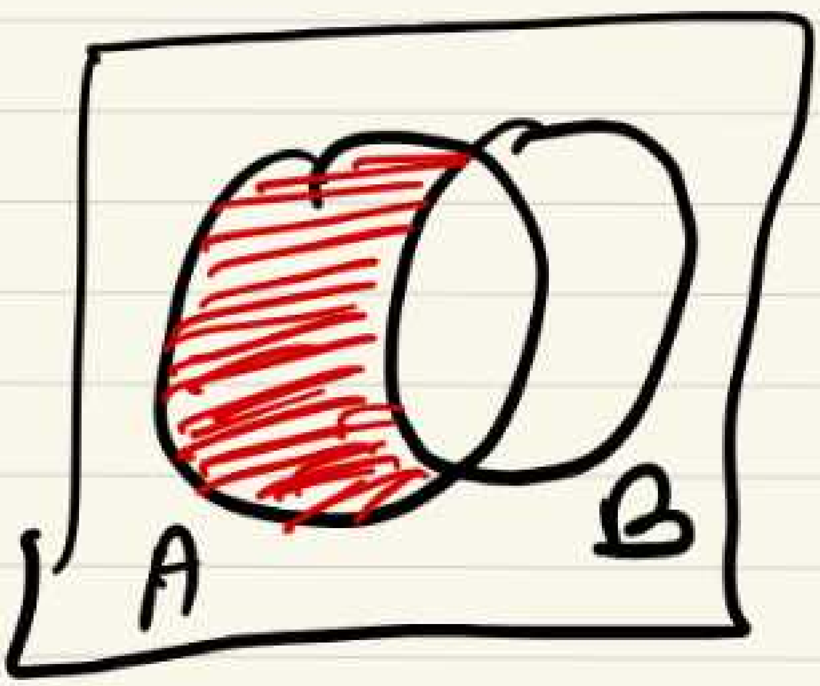
$P(A \text{ but not } B)$

$= P(\text{only } A)$

$= P(A - B)$

$= P(A \cap \bar{B})$

$= P(A) - P(A \cap B)$



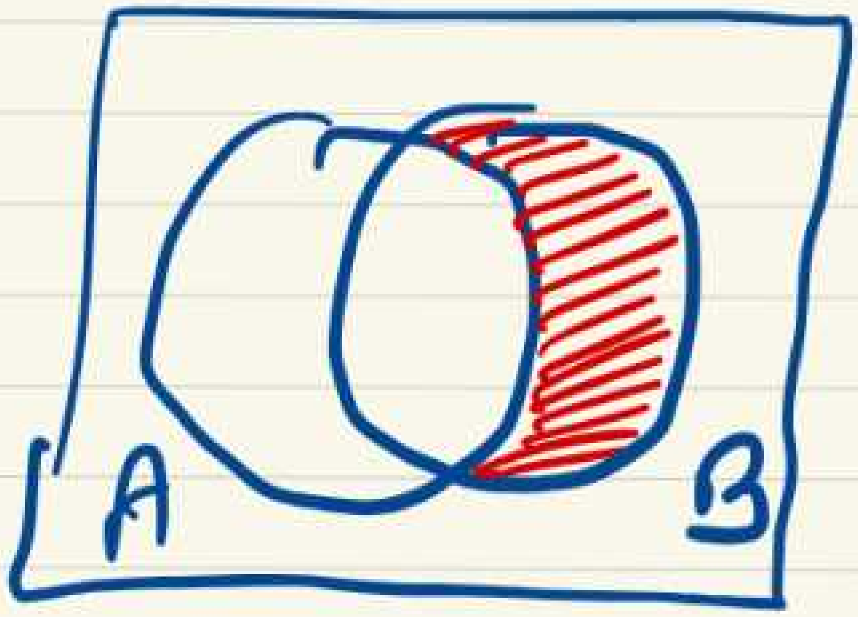
$P(B \text{ but not } A)$

$= P(\text{only } B)$

$= P(B - A)$

$= P(B \cap \bar{A})$

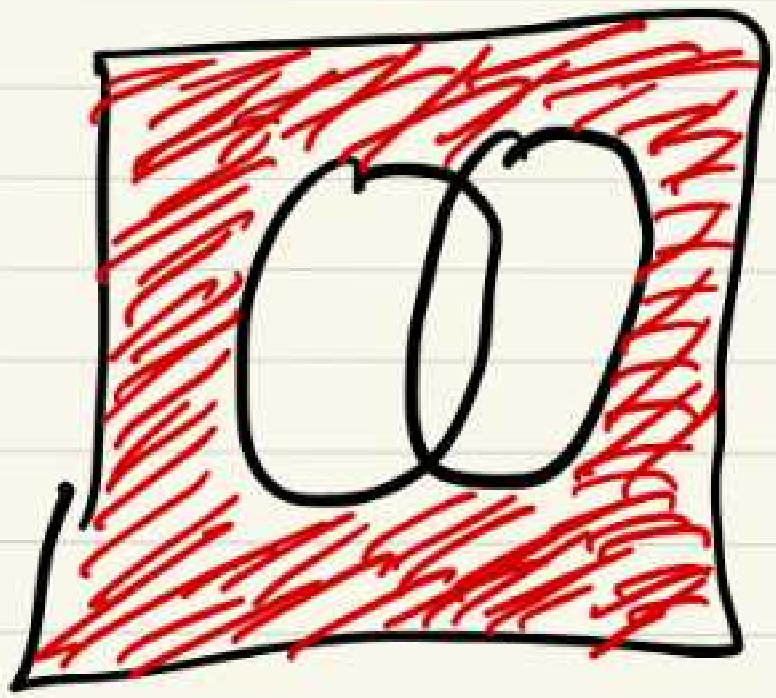
$= P(B) - P(A \cap B)$



P(Neither A Nor B)

$= P(A' \cap B') = P(A \cup B)'$

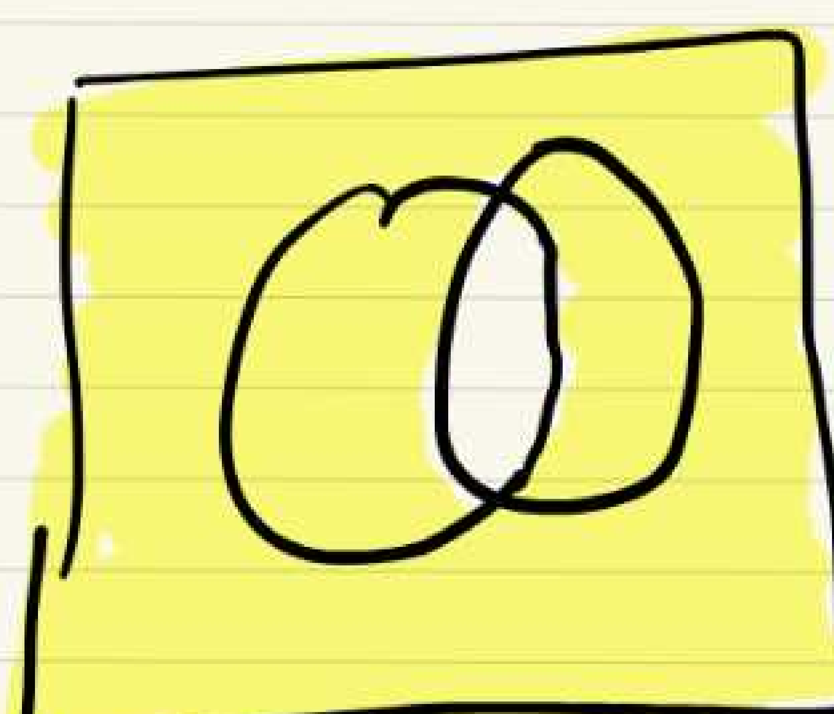
$= 1 - P(A \cup B)$



P(Not A or Not B)

$= P(A' \cup B') = P(A \cap B)'$

$= 1 - P(A \cap B)$



Conditional Probability

$P(A/B)$ = Prob. of event of A when event B has already occurred

$$\boxed{P(A/B) = \frac{P(A \cap B)}{P(B)}}$$ where $P(B) \neq 0$

$\rightarrow P(A/\bar{B}) = \dfrac{P(A \cap \bar{B})}{P(\bar{B})}$, $P(\bar{B}) \neq 0$

$\rightarrow P(\bar{A}/\bar{B}) = \dfrac{P(\bar{A} \cap \bar{B})}{P(\bar{B})}$, $P(\bar{B}) \neq 0$

Compound Probability Theorem

$$P(A \cap B) = P(A) \times P(B/A)$$

$$P(A \cap B \cap C) = P(A) \times P(B/A) \times P(C/A \cap B)$$

for independent events

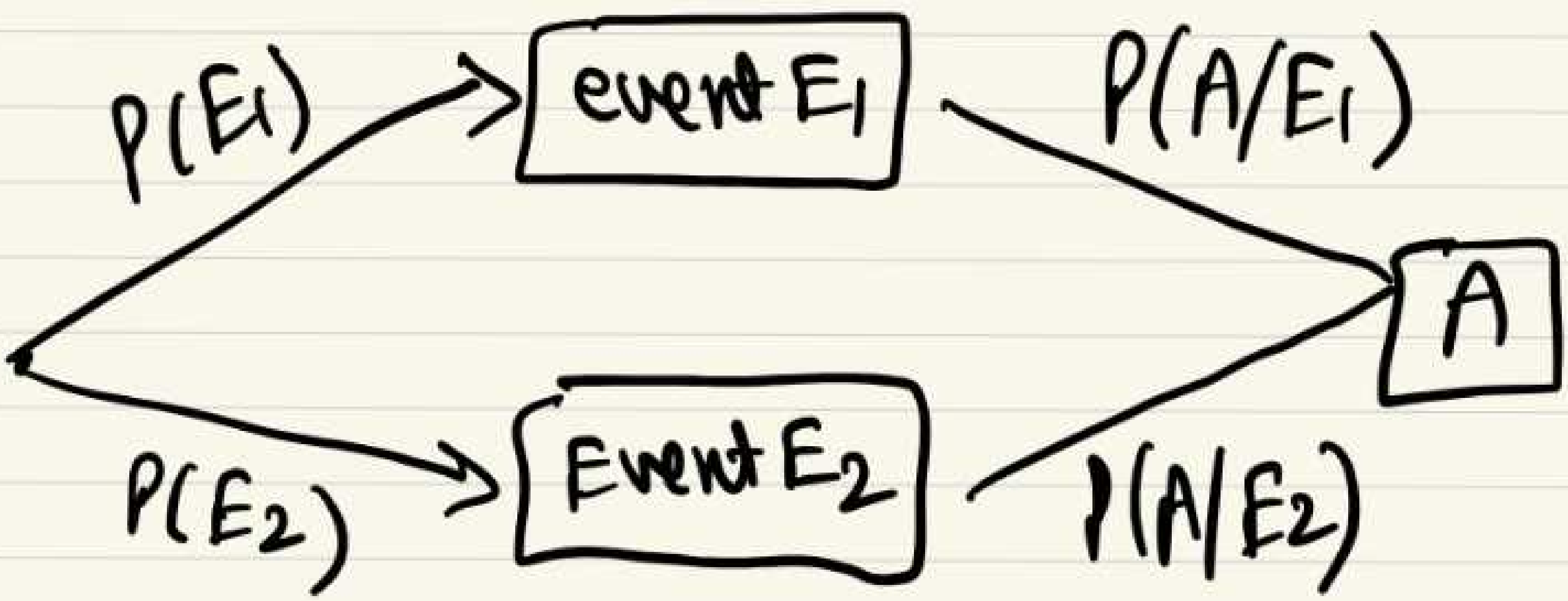
A & B are independent

iff $\boxed{P(A \cap B) = P(A) \times P(B)}$

or

$$P(A/B) = P(A) \;\&\; P(B/A) = P(B)$$

Total Probability Theorem



$$P(A) = P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2)$$

Random variable & Probability Distribution

Random variable (x_i) : variable on the basis of which probability is distributed

eg No of Heads, Age, weight

Random variable

Descrete
⇓
Probability mass function

continous
⇓
Probability Density function

x_i	p_i	p_ix_i	$p_ix_i^2$
	1	Σp_ix_i	$\Sigma p_ix_i^2$

$$\Sigma p_i = 1$$
(always)

Mean (μ) = $E(x) = \Sigma p_ix_i$
(expectation)

variance = $\sigma^2 = \Sigma p_i(x_i - \mu_i)^2$

or

$\Sigma p_ix_i^2 - (\Sigma p_ix_i)^2$

or

$E(x^2) - [E(x)]^2$

S.D. $= \sqrt{\text{variance}}$

for a constent K

$E(K) = K$

$E(x+y) = E(x) + E(y)$

$E(Kx) = K\,E(x)$

If x & y are independent

$E(x \cdot y) = E(x) \cdot E(y)$